4.6 Applying the Exponent Laws

LESSON FOCUS Apply the exponent laws to simplify expressions.

Make Connections

Recall the exponent laws for integer bases and whole number exponents.

Product of powers:	$a^m \cdot a^n = a^{m+n}$
Quotient of powers:	$a^m \div a^n = a^{m-n}$, $a \neq 0$
Power of a power:	$(a^m)^n = a^{mn}$
Power of a product:	$(ab)^m = a^m b^m$
Power of a quotient:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

please copy

What other types of numbers could be a base? An exponent?
How would you use the exponent laws to evaluate an expression with these numbers?

1. Product of powers: $a^m \cdot a^n = a^{m+n}$
2. Quotient of powers: $a^m \div a^n = a^{m-n}$, $a \neq 0$
3. Power of a power: $(a^m)^n = a^{mn}$
4. Power of a product: $(ab)^m = a^m b^m$
5. Power of a quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

Ex:

#1) $2^3 \cdot 2^5 = 2^{3+5} = 2^8$

#2) $\frac{a^5}{a^{-3}} = a^{5-(-3)} = a^8$

#3 $(3^2)^{\frac{2}{3}} = 3^{2 \times \frac{2}{3}} = 3^{4/3}$

#4 $(a^2 b^3)^4 = a^{2\times4} b^{3\times4} = a^8 b^{12}$

#5 $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$

Please try all of the following:

Q Identify and explain the errors in the following:

1st set (whole number exponents):

#1 a) $4^3 + 4^2 = 4^5$ b) $\frac{x^6}{x^3} = x^2$ c) $(10^2)^5 = 10^7$ d) $\left(\frac{1}{4}\right)^2 = \frac{1}{8}$

e) $(x-y)^3 = 3x - 3y$ f) $3^5 \times 3^2 = 3^{10}$ g) $5^3 \div 5^4 = \frac{3}{4}$

2nd set (integral exponents):

#2 a) $a^4 \bullet a^{-2} = a^{-8}$ b) $b^{-10} \div b^5 = b^{-5}$ c) $(c^{-3})^2 = c^{-1}$ d) $\left(\frac{2}{3}\right) - 2 = \frac{-4}{-6}$

3rd set (rational exponents):

#3 a) $2^{\frac{1}{2}} \bullet 2^{\frac{1}{2}} = 2^{\frac{1}{4}}$ b) $3^{\frac{3}{4}} \div 3^{\frac{1}{4}} = 3^3$ c) $\left(4^{\frac{2}{5}}\right)^2 = 4^{\frac{2}{25}}$ d) $\left(5^{\frac{1}{2}}\right)^2 = 5^{\frac{1}{2^2}}$

Q Fill in the blanks:

#4 a) $5^{-2} = \frac{\square}{\square}$ b) $6^{\square} = \frac{1}{6^2}$ c) $\square^{-6} = \frac{1}{10^6}$ d) $4^{-x} = \frac{1}{\square}$

Q Solve the following, by substituting the values given:

#5 a) $5x^4 + 6xy \quad if \quad x = 2, \; y = 3$

b) $(2x)^2 \quad if \; x = 4$

c) $(t+s)^{-3} \quad if \; t = 2, \; s = 4$

Show your work

Q Indicate if the following statements are always true, sometimes true, or false. Justify your answer.

a) The value of a power with a negative exponent is less than 0.

b) The value of a power for which the base is a fraction is less than 1.

c) Two powers with the exponent 0 have the same value.

Q During an exam, three students evaluate $2^{-2} \times 2^0$ as follows:

Thomas: $2^{-2} \times 2^0 = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

Sean: $2^{-2} \times 2^0 = 2^0 = 1$

Michel: $2^{-2} \times 2^0 = 4^0 = 1$

a) Identify the errors made by the students.

b) What is the correct answer? Justify your answer by explaining each step.

Act In pairs explain how to evaluate powers such as $(-3)^{-2}$ and -3^{-2}. Compare your answers with other groups, and then as a class.

THINK ABOUT IT

Work on your own.

What is the value of $\left(\frac{a^6b^9}{a^5b^8}\right)^{-2}$ when $a = -3$ and $b = 2$?

Compare strategies with a classmate.
If you used the same strategy, find a different strategy.
Which strategy is more efficient, and why?

$\left(\frac{a^6b^9}{a^5b^8}\right)^{-2}$

$a = -3$ and $b = 2$?

Simplify by writing as a single power. Explain the reasoning.

a) $0.3^{-3} \cdot 0.3^{5}$

$= 0.3^{2}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

$\left(-\frac{3}{2}\right)^{-8} \cdot \left(-\frac{3}{2}\right)^{6} = \left(-\frac{3}{2}\right)^{-2}$

$= \left(-\frac{2}{3}\right)^{2}$

c) $\frac{(1.4^3)(1.4^4)}{1.4^{-2}} =$

$3+4-(-2)$

$= 1.4^{9}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^{6}$

CHECK YOUR UNDERSTANDING

SOLUTION

Example 1 Simplifying Numerical Expressions with Rational Number Bases

Simplify by writing as a single power. Explain the reasoning.

a) $0.3^{-3} \cdot 0.3^{5}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

c) $\frac{(1.4^{3})(1.4^{4})}{1.4^{-2}}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^{6}$

SOLUTION

a) $0.3^{-3} \cdot 0.3^{5}$

Use the product of powers law:

When the bases are the same, add the exponents.

$0.3^{-3} \cdot 0.3^{5} = 0.3^{(-3)+5}$

$= 0.3^{2}$

(Solution continues.)

Example 1 Simplifying Numerical Expressions with Rational Number Bases

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

First use the power of a power law:

For each power, multiply the exponents.

$\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} = \left(-\frac{3}{2}\right)^{(-4)(2)} \cdot \left(-\frac{3}{2}\right)^{(2)(3)}$

Then use the product of powers law.

$\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} = \left(-\frac{3}{2}\right)^{-8} \cdot \left(-\frac{3}{2}\right)^{6}$

$= \left(-\frac{3}{2}\right)^{-2}$ Write with a positive exponent.

$= \left(-\frac{2}{3}\right)^{2}$

(Solution continues.)

4.6 Applying the Exponent Laws

Example 1 Simplifying Numerical Expressions with Rational Number Bases

c) $\dfrac{(1.4^3)(1.4^4)}{1.4^{-2}}$ Use the product of powers law.

$= \dfrac{1.4^{3+4}}{1.4^{-2}}$

$= \dfrac{1.4^7}{1.4^{-2}}$ Use the quotient of powers law.

$= 1.4^{7-(-2)}$

$= 1.4^9$

d) $\left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^6$ Use the product of powers law.

$= \left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{1}{3}+\frac{5}{3}}}\right)^6$

$= \left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}}\right)^6$ Use the quotient of powers law.

(Solution continues.)

Example 1 Simplifying Numerical Expressions with Rational Number Bases

$= \left(7^{\frac{2}{3}-\frac{6}{3}}\right)^6$

$= \left(7^{-\frac{4}{3}}\right)^6$ Use the power of a power law.

$= 7^{\left(-\frac{4}{3}\right)(6)}$

$= 7^{-\frac{24}{3}}$

$= 7^{-8}$ Write with a positive exponent.

$= \dfrac{1}{7^8}$

CHECK YOUR UNDERSTANDING

Example 2 Simplifying Algebraic Expressions with Integer Exponents

Simplify. Explain the reasoning.

a) $(x^3y^2)(x^2y^{-4})$ b) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

SOLUTION

CHECK YOUR UNDERSTANDING

Example 2 Simplifying Algebraic Expressions with Integer Exponents

Simplify. Explain the reasoning.

a) $(x^3y^2)(x^2y^{-4})$ b) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

SOLUTION

a) $(x^3y^2)(x^2y^{-4}) = x^3 \cdot y^2 \cdot x^2 \cdot y^{-4}$ x^3y^2 means $x^3 \cdot y^2$

$= x^3 \cdot x^2 \cdot y^2 \cdot y^{-4}$ Use the product of powers law.

$= x^{3+2} \cdot y^{2+(-4)}$

$= x^5 \cdot y^{-2}$ Write with a positive exponent.

$= x^5 \cdot \dfrac{1}{y^2}$

$= \dfrac{x^5}{y^2}$

b) $\dfrac{10a^5b^3}{2a^2b^{-2}} = \dfrac{10}{2} \cdot \dfrac{a^5}{a^2} \cdot \dfrac{b^3}{b^{-2}}$ Use the quotient of powers law.

$= 5 \cdot a^{5-2} \cdot b^{3-(-2)}$

$= 5 \cdot a^3 \cdot b^5$

$= 5a^3b^5$

CHECK YOUR UNDERSTANDING

Example 3 Simplifying Algebraic Expressions with Rational Exponents

Simplify. Explain the reasoning.

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

CHECK YOUR UNDERSTANDING

SOLUTION

Example 3 Simplifying Algebraic Expressions with Rational Exponents

Simplify. Explain the reasoning.

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

SOLUTION

a) $(8a^3b^6)^{\frac{1}{3}} = 8^{\frac{1}{3}} \cdot a^{3\left(\frac{1}{3}\right)} \cdot b^{6\left(\frac{1}{3}\right)}$ Using the power of a power law.

$= (2^3)^{\frac{1}{3}} \cdot a^1 \cdot b^2$

$= 2ab^2$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) = x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} \cdot y^2 \cdot y^{-1}$ Use the product of powers law.

$= x^{\frac{3}{2}+\frac{1}{2}} \cdot y^{2+(-1)}$

$= x^2y$

(Solution continues.)

4.6 Applying the Exponent Laws

Example 3 Simplifying Algebraic Expressions with Rational Exponents

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^{2}b^{\frac{1}{3}}} = \dfrac{4}{2} \cdot \dfrac{a^{-2}}{a^{2}} \cdot \dfrac{b^{\frac{2}{3}}}{b^{\frac{1}{3}}}$ Use the quotient of powers law.

$= 2 \cdot a^{(-2)-2} \cdot b^{\frac{2}{3}-\frac{1}{3}}$

$= 2 \cdot a^{-4} \cdot b^{\frac{1}{3}}$ Write with a positive exponent.

$= \dfrac{2b^{\frac{1}{3}}}{a^{4}}$

(Solution continues.)

Example 3 Simplifying Algebraic Expressions with Rational Exponents

d) $\left(\dfrac{100a}{25a^{5}b^{-\frac{1}{2}}}\right)^{\frac{1}{2}} = \left(\dfrac{100}{25} \cdot \dfrac{a^{1}}{a^{5}} \cdot \dfrac{1}{b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$ Simplify inside the brackets first. Use the quotient of powers law. Write with a positive exponent.

$= \left(4 \cdot a^{1-5} \cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= \left(4 \cdot a^{-4} \cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$ Use the power of a power law.

$= 4^{\frac{1}{2}} \cdot a^{(-4)\left(\frac{1}{2}\right)} \cdot b^{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$

$= 2 \cdot a^{-2} \cdot b^{\frac{1}{4}}$ Write with a positive exponent.

$= \dfrac{2b^{\frac{1}{4}}}{a^{2}}$

CHECK YOUR UNDERSTANDING

Please copy the following, we are going to do them together.

a) $(x^3y^2)(x^2y^{-4})$

x^5y^{-2}

b) $\frac{10a^5b^3}{2a^2b^{-2}} = 5a^3b^5$

$3-(-2)$

c) $\frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}} = 2a^{-4}b^{\frac{1}{3}}$

$-2-2$

$\frac{2}{3}-\frac{1}{3}$

$-\frac{4}{3}\cdot\frac{6}{1} = -\frac{24}{3}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}}\cdot 7^{\frac{3}{3}}}\right)^6$

$\left(\frac{7^{2/3}}{7^{4/3}}\right)^6$

$= \left(7^{-\frac{4}{3}}\right)^6 = 7^{-8}$

Review Assignment

It will be marked for completeness. That means that you will have to pass it in at the beginning of class tomorrow. I will look it over to make sure the you made an effort to complete the assignment and try each question. Based on how well I believe you completed the assignment, I will give you a mark out of 20.

You have the rest of today's class to work on completing the assignment. Make sure use your AP class to work on finishing up the assignment. You may also have to do some for homework.

I am available at lunch hour today for extra help as well.